Name _____   Student Number _____

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions. Solve the following equations and inequalities. All answers should be given in interval notation.

(1) $|5x + 4| = |3x + 2|$

$$5x + 4 = 3x + 2 \text{ and } 5x + 4 = -(3x + 2)$$

$$x = -1 \text{ and } x = -\frac{3}{4}$$

(2) $2x - 3 \geq \dfrac{2}{x}$

$$2x - 3 - \frac{2}{x} \geq 0$$

$$\frac{2x^2 - 3x - 2}{x} \geq 0$$

$$\frac{(2x + 1)(x - 2)}{x} \geq 0$$

Use a sign table:

	$x < -\frac{1}{2}$	$-\frac{1}{2} < x < 0$	$0 < x < 2$	$x > 2$
$2x + 1$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
$x - 2$	$-$	$-$	$-$	$+$
$\frac{(2x+1)(x-2)}{x}$	$-$	$+$	$-$	$+$

The solution set is

$$x \in [-\frac{1}{2}, 0) \cup [2, \infty)$$

Note that 0 is not included, since it would make the equation undefined.

Over→

(3) $|x^2 + 1| > 5$

$$x^2 + 1 > 5 \;\; \text{or} \;\; x^2 + 1 < -5$$

The first equation gives

$$x^2 > 4$$
$$|x| > 2$$
$$x > 2 \;\; \text{or} \;\; x < -2$$

The second equation gives

$$x^2 < -6$$

which has no solution, since x^2 is always a positive number. Therefore, the solution set is

$$x \in (-\infty, -2) \cup (2, \infty)$$

(4) $2x^2 - 1 < -3x$

$$2x^2 + 3x - 1 < 0$$

To get the sign table, we need to factor this equation. It does not factor nicely, so use the quadratic equation to find the solutions to $2x^2 + 3x - 1 = 0$.

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2 \cdot 2}$$
$$x = \frac{-3 \pm \sqrt{17}}{4}$$

So, we're solving

$$\left(x - \left(\frac{-3 - \sqrt{17}}{4} \right) \right) \left(x - \left(\frac{-3 + \sqrt{17}}{4} \right) \right) < 0$$

Write

$$a_1 = \left(\frac{-3 - \sqrt{17}}{4} \right)$$
$$a_2 = \left(\frac{-3 + \sqrt{17}}{4} \right)$$

Use a sign table:

	$x < a_1$	$a_1 < x < a_2$	$x > a_2$
$x - a_1$	$-$	$+$	$+$
$x - a_2$	$-$	$-$	$+$
$(x - a_1)(x - a_2)$	$+$	$-$	$+$

So, the solution set is

$$x \in (a_1, a_2)$$

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